Algorithms for stochastic nonconvex and nonsmooth optimization

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Research Directions

(1). Thesis Work

- computational complexity
- nonsmooth analysis of eigenvalues
- composite nonlinear models $(h \circ c)$
- statistical guarantees for nonconvex problems

(2). Post doc

- stochastic optimization
- constrained conjugate gradient

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(1). Local search for non-smooth and non-convex problems
(2). Adaptive line search for stochastic optimization

$\boxed{\text{Local search for non-smooth and non-convex problems}}$

Joint work with D. Davis (Cornell), D. Drusvyatskiy (U. Washington), and K. MacPhee (U. Washington)

Why study nonsmooth and nonconvex optimization?

$$\min_x \; g(x)$$

Nonsmooth and nonconvex losses arise often...

- Structure (sparsity), robustness (outliers), stability (better conditioning)

Common problem class: (convex) $\circ$ (smooth)

(Fletcher '80, Powell '83, Burke '85, Wright '90, Lewis-Wright '08, Cartis-Gould-Toint '11)

Example: Robust Phase Retrieval

Problem: Find $x \in \mathbb{R}^d$ such that

$$(a_i^T x)^2 \approx b_i \quad a_1, \ldots, a_m \in \mathbb{R}^d, \quad b_1, \ldots, b_m \in \mathbb{R}.$$

Composite formulation:

$$\min_x \; g(x) := \frac{1}{m} \sum_{i=1}^{m} |(a_i^T x)^2 - b_i|$$

Assumptions:

$$m \gtrsim d, \quad b_i = (a_i^T \bar{x})^2 \text{ for some } \bar{x} \in \mathbb{R}^d, \quad a_i \sim N(0, I_d) \text{ and independent}$$

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Consequences: $\exists$ constants $\rho, \tau > 0$ such that with probability $1 - e^{-cm}$

- **Weakly-convex**: (Duchi-Ruan '17)

$$x \mapsto g(x) + \frac{\rho}{2} \|x\|_2^2 \quad \text{is convex}$$

- **Sharpness**: (Eldar-Mendelson '14)

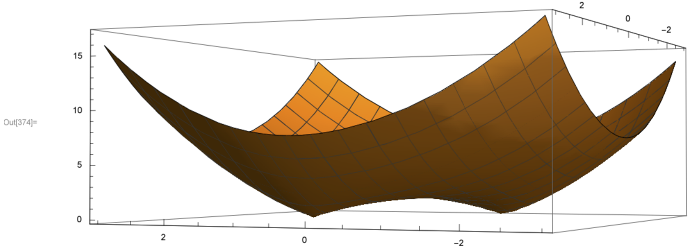
$$g(x) \geq \tau \|\bar{x}\|_2 \operatorname{dist}(x, \{\pm \bar{x}\}).$$

Holds even when up to 1/2 the points are corrupted!

Intuition

$g(x) = \frac{1}{m}\sum_{i=1}^{m} |(a_i^T x)^2 - b_i|$ approximates the population objective:

$$g_P(x) = \mathbf{E}_{a\sim N}[|(a^T x)^2 - (a^T \bar{x})^2|]$$



Local search

$$\min_x \; g(x)$$

Strategy:

- Find a moderately accurate solution $\hat{x}$ at a low sample complexity cost
  - Available for: phase retrieval (Candès et al. '15), blind deconvolution (Ma et al. '17, Li et al. '18), matrix sensing (Boczar et al. '16)
- Refine $\hat{x}$ with a rapidly converging algorithm

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g is μ-sharp and ρ-weakly convex

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Is there a generic gradient-based local search procedure for nonsmooth and nonconvex problems?

Stationary points

- Subdifferential $\partial g(x)$ consists of $v \in \mathbb{R}^d$ such that

$$g(y) \geq g(x) + \langle v, y - x \rangle + o(\|y - x\|), \quad \text{as } y \to x$$

- Under weak-convexity,

$$x \text{ stationary} \quad \Leftrightarrow \quad \inf_{\|v\|=1} g'(x, v) \geq 0 \quad \Leftrightarrow \quad 0 \in \partial g(x).$$

Remark:

$$\boxed{\partial(h \circ c)(x) = \nabla c(x)^T \partial h(c(x))}$$

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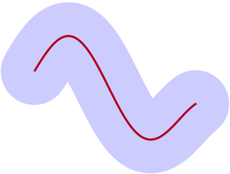
Lemma (Davis-Drusvyatskiy-MacPhee-**P**)
No extraneous stationary points of g lie in the tube:

$$\boxed{\mathcal{T} := \left\{ x \in \mathbb{R}^d \; : \; \mathrm{dist}(x; S) < \frac{\boldsymbol{\mu}}{\boldsymbol{\rho}} \right\}}$$



Meta-Theorem:

Simple algorithms for sharp and weakly convex functions converge rapidly

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Simple algorithms for sharp and weakly convex functions converge rapidly

$$\min_x \; g(x) := h(c(x)), \quad \text{(convex)} \circ \text{(smooth)}$$

Prox-linear method:

$$x^+ := \operatorname*{argmin}_y \left\{ h\big(\boldsymbol{c(x) + \nabla c(x)(y - x)}\big) + \frac{\rho}{2}\|y - x\|^2 \right\}$$

(Burke '85, '91, Fletcher '82, Powell '84, Wright '90, Yuan '83, Cartis-Gould-Toint '11)

Eg: proximal gradient, Levenberg-Marquardt

Subgradient method:

$$x^+ = x - \left(\frac{g(x) - \inf\, g}{\|v\|^2}\right) v \quad \text{where } v \in \partial g(x)$$

Prox-linear method

$$\boxed{x^+ = \operatorname*{argmin}_y \ \{h(c(x) + \nabla c(x)(y - x)) + \tfrac{\rho}{2}\|y - x\|^2\}}$$

Thm: (Burke-Ferris '93)

Suppose that $g = h \circ c$ is

- ρ-weakly convex
- μ-sharp
- $\mathrm{dist}(x_0, S) \leq \frac{\mu}{2\rho}$

Then

$$\mathrm{dist}(x_{k+1}, S) \leq \frac{\mu}{\rho} \cdot \left(\frac{1}{2}\right)^{2^k} \qquad \text{for all } k.$$

Global convergence guarantees (Drusvyatskiy-**P**, Math. Program '16)

Eg: phase retrieval

- $\frac{\mu}{\rho}$ is **dimension independent** w.h.p. (Eldar-Mendelson '14, Duchi-Ruan '17)

Subgradient Methods

Polyak subgradient method:

$$x^+ = x - \left(\frac{g(x) - \inf\ g}{\|v\|^2}\right) v \quad \text{where } v \in \partial g(x).$$

Thm: (Polyak '67, Davis-Drusvyatskiy-MacPhee-**P** '17)

Suppose that g is

- ρ-weakly convex
- L-Lipschitz
- μ-sharp
- $\mathrm{dist}(x_0, S) \leq \frac{\mu}{2\rho}$

Then

$$\frac{\mathrm{dist}(x_{k+1}, S)}{\mathrm{dist}(x_k, S)} \leq \sqrt{1 - \left(\frac{\mu}{L\sqrt{2}}\right)^2}, \qquad \text{for all } k.$$

Eg: phase retrieval

- $\frac{\mu}{\rho}$, $\frac{\mu}{L}$ are **dimension independent** w.h.p. (Eldar-Mendelson '14, Davis-Drusvyatskiy-MacPhee-**P** '17)

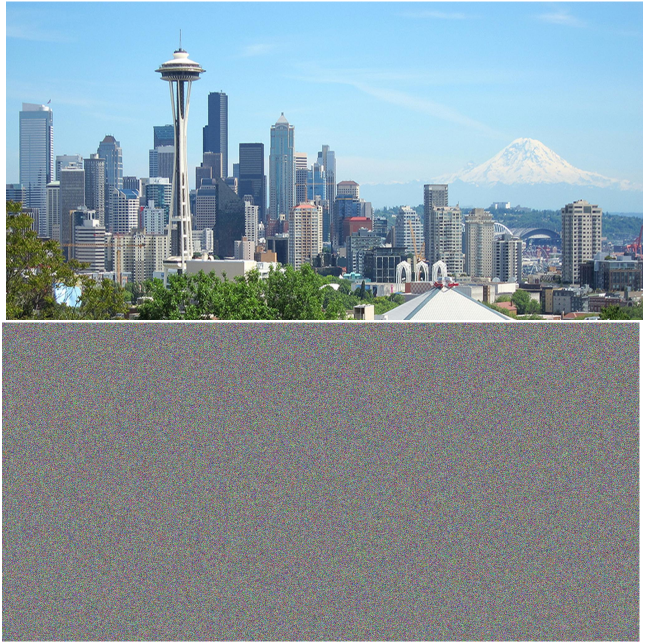

Figure: $(d, m) \approx (2^{23}, 2^{24})$. Iteration 1.

Figure: $(d, m) \approx (2^{23}, 2^{24})$. Iteration 2.

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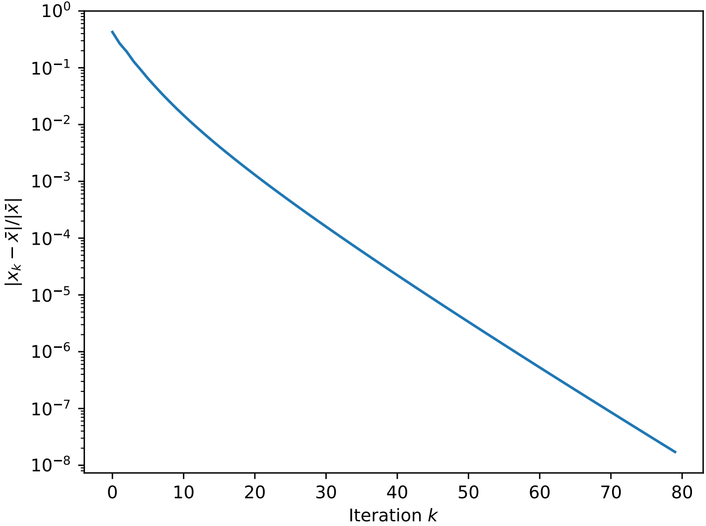

Figure: Convergence plot (iterates vs. $\|x_k - \bar{x}\|/\|\bar{x}\|$).

Research Program:

Physical/Statistical well-posedness	$\Rightarrow$	(convex) $\circ$ (smooth) with "sharpness"-like conditions	$\Rightarrow$	Simple algorithms converge rapidly

- Robust phase retrieval (Duchi-Ruan '17, Davis-Drusvyatskiy-MacPhee-P '17)
- Blind deconvolution/bi-convex sensing (Ling-Strohmer '15, Ahmed et al. '14)
- Covariance estimation (Chen et. al '15)
- Robust PCA (Candes et al. '11, Chandrasekaran et al. '11, Netrapalli et al. '14)
- Non-negative matrix factorization (Lee-Seung '99, Donoho-Stodden '03)

Adaptive line search method
for smooth stochastic optimization

Joint work with K. Scheinberg

Stochastic optimization

Central task in machine learning

$$\min_x \; f(x) = \mathbf{E}_{\xi \sim P}[\tilde{f}(x;\xi)]$$

Stochastic gradient descent (SGD):

$$\left\{ \begin{array}{l} \text{Pick } \xi \sim P \\ \text{Set } x_{k+1} = x_k - \alpha_k \nabla \tilde{f}(x_k;\xi) \end{array} \right\}$$

- **Major drawback:** stepsize, α_k, requires lots of tuning

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Deterministic setting: Use line search techniques

Question:
Can the line search technique be adapted to the stochastic setting?

(Deterministic) Backtracking Line Search

Classical problem

$$\min_{x \in \Omega} \; f(x)$$

$f : \Omega \to \mathbf{R}$ with L-Lipschitz gradient

Gradient descent: $x_{k+1} = x_k - \alpha \nabla f(x_k), \quad \alpha \in (0, 1/L]$

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Backtracking Line Search Algorithm

- Compute $f(x_k)$ and $\nabla f(x_k)$
- Check sufficient decrease (Armijo '66)

$$\boxed{f(x_k - \alpha_k \nabla f(x_k)) \leq f(x_k) - \theta \alpha_k \left\| \nabla f(x_k) \right\|^2}$$

- Successful: $x_{k+1} = x_k - \alpha_k \nabla f(x_k)$ and increase $\alpha_k \Rightarrow \alpha_{k+1} = \gamma^{-1} \alpha_k$
- Unsuccessful: $x_{k+1} = x_k$ and decrease $\alpha_k \Rightarrow \alpha_{k+1} = \gamma \alpha_k$

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$$\boxed{\underbrace{f(x_k - \alpha_k \nabla f(x_k))}_{\text{function value at next step}} \leq \underbrace{f(x_k) - \theta \alpha_k \|\nabla f(x_k)\|^2}_{\text{linearization of } f \text{ at current step}}}$$

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Question

Can the line search technique be adapted to stochastic setting using only knowable quantities?

Knowable quantities: e.g. bound on variance of $\nabla \tilde{f}$, $\tilde{f}$

Related works

Line search & heuristics Previous work requires: $\nabla f(x)$, $\alpha_k \to 0$

- Bollapragada, Byrd, and Nocedal; "Adaptive sampling strategies for stochastic optimization" (to appear in SIOPT 2017)
- Friedlander and Schmidt; "Hybrid deterministic-stochastic methods for data fitting" (2012, SIAM Sci. Comput)
- Mahsereci and Hennig; "Probabilistic line search for stochastic optimization" (JMLR 2018; NIPS 2015)

Stochastic backtracking line search

- Compute stochastic estimates $\underbrace{g_k}_{\nabla f(x_k)}$, $\underbrace{f_k}_{f(x_k)}$, and $\underbrace{f_k^+}_{f(x_k - \alpha_k g_k)}$
- Check sufficient decrease (Armijo '66)

$$f_k^+ \leq f_k - \theta \alpha_k \|g_k\|^2$$

- Successful: $x_{k+1} = x_k - \alpha_k g_k$ and increase $\alpha_k \Rightarrow \alpha_{k+1} = \gamma^{-1} \alpha_k$
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Challenges

$$f_k^+ \leq f_k - \theta \alpha_k \|g_k\|^2 \quad \overset{??}{\Rightarrow} \quad f(x_k - \alpha_k g_k) \leq f(x_k) - \theta \alpha_k \|\nabla f(x_k)\|^2$$

- $\boxed{\text{Bad function estimates may} \uparrow \text{objective value}}$

  Increase at most $\alpha_k^2 \|g_k\|^2$

- Stepsizes, α_k, become arbitrarily small

Stochastic line search

Algorithm

- Compute random estimate of the gradient, g_k
- Compute random estimate of $f_k \approx f(x_k)$ and $f_k^+ \approx f(x_k - \alpha_k g_k)$
- Check the stochastic sufficient decrease

$$f_k^+ \leq f_k - \theta \alpha_k \left\| g_k \right\|^2$$

- Successful: $x_{k+1} = x_k - \alpha_k g_k$ and $\alpha_k \uparrow \Rightarrow \alpha_{k+1} = \gamma^{-1} \alpha_k$

  - Reliable step: If $\alpha_k \left\| g_k \right\|^2 \geq \delta_k^2$, $\uparrow \delta_k \Rightarrow \delta_{k+1}^2 = \gamma^{-1} \delta_k^2$
  - Unreliable step: If $\alpha_k \left\| g_k \right\|^2 < \delta_k^2$, $\downarrow \delta_k \Rightarrow \delta_{k+1}^2 = \gamma \delta_k^2$

- Unsucessful: $x_{k+1} = x_k$, decrease α_k, and $\boxed{\text{decrease } \delta_k}$
  $\Rightarrow \alpha_{k+1} = \gamma \alpha_k$ and $\delta_{k+1}^2 = \gamma \delta_k^2$.

Randomness assumptions

- **Accurate gradient g_k w/ prob. p_g:**

$$\mathbf{Pr}(\|g_k - \nabla f(x_k)\| \leq \alpha_k \|g_k\|) \geq p_g$$

- **Accurate function estimates f_k and f_k^+ w/ prob. p_f:**

$$\begin{aligned} \mathbf{Pr}(&|f(x_k) - f_k| \leq \alpha_k^2 \|g_k\|^2 \\ &\text{and} \qquad |f(x_k - \alpha_k g_k) - f_k^+| \leq \alpha_k^2 \|g_k\|^2) \geq p_f \end{aligned}$$

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- **Variance condition**

$$\mathbf{E}[|f_k - f(x_k)|^2] \leq \theta^2 \delta_k^4 \qquad \text{(same for } f_k^+\text{).}$$

Question: How to choose these probabilities (p_f, p_g) large enough?

$p_f, p_g \geq 1/2$ at least, but p_f should be large.

Satisfying randomness assumptions

$$\min_x \; f(x) = \mathbf{E}_{\xi \sim P}[\tilde{f}(x; \xi)]$$

and bound on variance

$$\mathbf{E}_{\xi \sim P}(\|\nabla \tilde{f}(x, \xi) - \nabla f(x)\|^2) \leq V_g, \quad \mathbf{E}_{\xi \sim P}(|\tilde{f}(x; \xi) - f(x)|^2) \leq V_f.$$

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Example: sampling

$$g_k = \frac{1}{|S_g|} \sum_{i \in S_g} \nabla \tilde{f}(x_k;\xi_i), \qquad f_k = \frac{1}{|S_f|} \sum_{i \in S_f} \tilde{f}(x_k;\xi_i).$$

How many samples do we need?

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$$\boxed{\text{How many samples do we need?}}$$

Chebyshev Inequality

$$|S_g| \approx \tilde{O}\left(\frac{V_g}{\alpha_k^2 \|g_k\|^2}\right), \qquad |S_f| \approx \tilde{O}\left(\max\left\{\frac{V_f}{\alpha_k^4 \|g_k\|^4}, \frac{V_f}{\delta_k^4}\right\}\right)$$

Convergence result

Key observations

- $\Phi_k = \underbrace{\nu(f(x_k) - \inf f) + (1-\nu)\alpha_k \|\nabla f(x_k)\|^2}_{\text{balance each other}} + (1-\nu)\theta{\delta_k}^2$
- $T_\varepsilon = \inf\{k \geq 0 \,:\, \|\nabla f(x_k)\| < \varepsilon\}$

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Thm: (P-Scheinberg '18) If $p_g p_f > 1/2$ and p_f sufficiently large, then

$$\mathbf{E}[\Phi_{k+1} - \Phi_k |\, \text{past}] \leq -\left(\alpha_k \|\nabla f(x_k)\|^2 + \theta\delta_k^2\right)$$

and consequently,

$$\mathbf{E}[T_\varepsilon] \leq \mathcal{O}\left(\frac{1}{\varepsilon^2}\right).$$

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Proof sketch

- α_k behave like random walk with $p = p_g p_f$
- $\{\Phi_k\}$ supermartingales with stopping time T_ε
- Optional stopping time and Wald's identity

Convex case

Assumptions:

- f is convex and $\|\nabla f(x)\| \leq L_f$ for all $x \in \Omega$
- $\|x - x^*\| \leq D$ for all $x \in \Omega$

Key observation:

$$\boxed{\Phi_k = \frac{1}{\nu\varepsilon} - \frac{1}{\Psi_k}}$$

where $\Psi_k = \nu(f(x_k) - \inf f) + (1-\nu)\alpha_k \|\nabla f(x_k)\|^2 + (1-\nu)\theta\delta_k^2$

Stopping time: $T_\varepsilon = \inf\{k \,:\, f(x_k) - \inf f < \varepsilon\}$

Convergence rate, convex (P-Scheinberg '18)

If $p_g p_f > 1/2$ and p_f sufficiently large, then

$$\mathbf{E}[T_\varepsilon] \leq \mathcal{O}\left(\frac{1}{\varepsilon}\right)$$

μ-strongly convex case

Key observation:

$$\boxed{\Phi_k = \log(\Psi_k) - \log(\nu\varepsilon)}$$

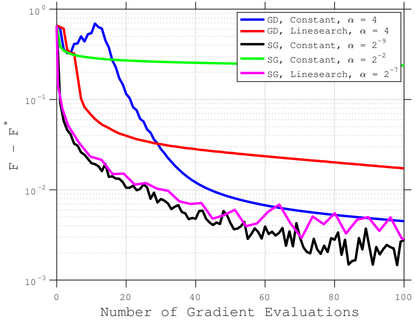
where $\Psi_k = \nu(f(x_k) - \inf f) + (1-\nu)\alpha_k \left\|\nabla f(x_k)\right\|^2 + (1-\nu)\theta\delta_k^2$

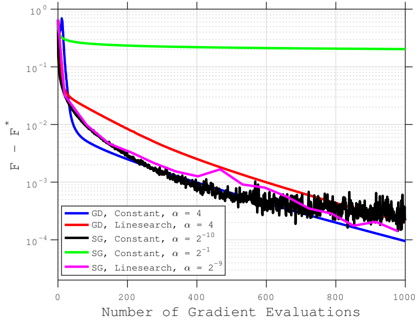
Convergence rate, strongly convex (P-Scheinberg '18)
If $p_g p_f > 1/2$ and p_f sufficiently large, then

$$\mathbf{E}[T_\varepsilon] \leq \mathcal{O}\left(\frac{L}{\mu}\log\left(\frac{1}{\varepsilon}\right)\right)$$

Preliminary results

$$\min_{\theta} \; \frac{1}{m} \sum_{i=1}^{m} \log(1 + \exp(-y_i(\theta^T x_i)) + \frac{\lambda}{2} \|\theta\|^2$$





Open questions and extensions

Conclusions

- General framework for convergence results
- Convergence analysis (nonconvex, convex, and strongly convex) for a line search algorithm with gradient descent.

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Applications of the stochastic process

- Line search, trust region methods (Blanchet, Cartis, Menickelly, Scheinberg '17), and cubic regularization?
- Extensions into 2nd order stochastic methods with Hessian guarantees?

Open problems

- Finding a good practical stochastic line search for machine learning; sampling procedure too conservative
- Extending line search procedure to stochastic Wolfe conditions (BFGS)

References

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